SOME ALGORITHMS FOR CONTROLLING THE MOTION OF SATELLITES IN A FORMATION

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In this paper the problem of studying the motion and maintaining of a tetrahedral formation of satellites in a geostationary orbit are considered.

To control the formation configuration in the case of an unperturbed reference orbit was developed algorithm on the base of root locus method. Moreover, to control of the formation configuration with respect to the unperturbed reference orbit algorithm based on feedback methods was developed. For this case it was decided to use a linear quadratic regulator, which is obtained by minimizing the quality criterion of the form.

Key Words: Spacecraft Formation, Geostationary Orbit, Tetrahedron Configuration, Earth Observation, Space Formation Control

1. Introduction

At present many researchers prefer to consider the satellite formation instead of single satellites for the problems of space exploration. It takes place due to the fact that the small satellites are usually used in the formations as the development of small satellites takes less time and money. In addition, with the help of formations it is possible to solve a whole class of new problems that are impossible to solve with one satellite. For example, the simultaneous measurement of any indicators at different spatial points, which is important when studying magnetic or gravitational fields, the ionosphere, atmosphere, etc. The possibilities of using the satellite formation for the problems of astronomical observations, stereographic survey of the Earth's surface, remote sensing of the Earth in real time are interesting. New opportunities of satellite formations force the arising of new problems. Control synthesis for maintaining and reconstruction of the satellite configuration in formation is one of the key problems for satellite formations. Various configurations are selected and in each case the control system is developed for the assigned problems depending on the mission[1,2]. To maintain the configuration of formation it is necessary to provide control over the relative motion of the satellites in the formation. This problem is the basis of the current paper. In this paper we consider the problem of studying the motion and maintaining of a tetrahedral formation of satellites in a geostationary orbit in the presence of disturbances caused by the influence of the inhomogeneity of the Earth's gravitational field.

2. Mathematical model

The formation is a system of four spacecraft forming a tetrahedron or regular pyramid. In this case, the main spacecraft is located above the three accompanying spacecrafts located in the plane of the tetrahedron base and moves in an orbit called the reference orbit (Figure 1). The accompanying spacecrafts form a regular triangle, and the height of the tetrahedron passes through the center of this triangle.

Let us derive the differential equations of motion of one the satellite in the formation relative to the reference satellite in the Earth's orbit. To do this, we denote the satellite under consideration as B and the reference satellite as A. Let's introduce the coordinate systems [3] shown in Figure 2:

OXYZ - inertial coordinate system (ICS), the center of which is located at the Earth's center of mass, the OZ axis is directed along the Earth's rotation axis, the OX is directed to the point of the Spring equinox of the J2000 epoch;

Axyz – local vertical local horizontal coordinate system (LVLH), the center of which is in the reference satellite, the Ax axis is directed along the radius vector of the reference satellite from the center of the Earth, the Az axis is normal to the orbit plane in the direction of the orbital momentum, the Ay axis completes the system to the right.

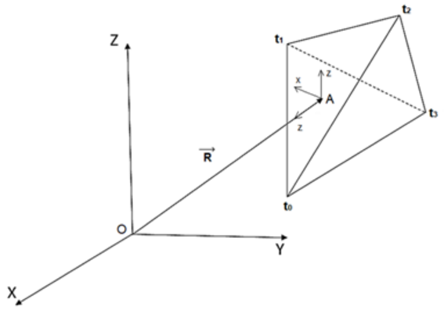

Fig. 1. Tetrahedral formation of satellites.

When considering the motion of satellites in the formation it is very important to choose the correct form of dynamics equations and coordinate system for development of control system keeping the configuration of the formation.

Differential equations of the formation motion on circular orbit have form [3]:

$$\ddot{x} - 2n\dot{y} - 3n^2 x = 0, \quad \ddot{y} + 2n\dot{x} = 0, \quad \ddot{z} + n^2 z = 0 \tag{1}$$

where $n = \mu / R^3$ is the mean motion, $\vec{\rho}(R+x, y, z)$ is radius vector of deputy satellite B, $\vec{R}(R,0,0)$ is radius vector of reference satellite A in the ICS and the radius vector $\vec{r}(x, y, z)$ of satellite B relative to the satellite A (Figure 2).

Equations (1) are linear differential equations of the relative motion of satellites in a formation in the case of an unperturbed circular reference orbit and are called the Hill-Clohessy-Wiltsher equations.

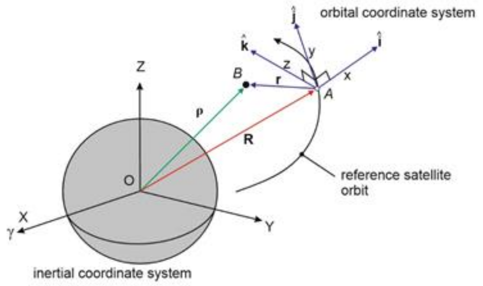

Fig. 2. Coordinate systems for describing the relative motion of satellites

For satellites in geostationary orbit it is necessary to take into account the influence of gravitational disturbances caused by the inhomogeneity of the Earth's gravitational field. To consider the influence of the figure of the Earth on the relative motion of the satellite in a formation described by equations (1), we obtain a perturbing function. To do this, we introduce the influence of the second zonal harmonic J_2 into the gravitational potential[2]:

$$U_2 = -\frac{1}{2}\mu J_2 \frac{r_E}{R^3}(3\sin^2\varphi - 1) \tag{2}$$

where r_E is the radius of the Earth, φ is the latitude of the deputy satellite in the ICS.

This expression can also be written relative to the OCS, taking $\varphi = \sin I \sin u$, where I is the inclination of the satellite orbit, u -is the argument of the satellite latitude.

Taking the gradient operation ∇ or the partial derivatives of the function described above, we determine the components of the gravitational force J_2 vector $\vec{J}_2(R) = \begin{bmatrix} f_x, & f_y, & f_z \end{bmatrix}$ and putting on the right side of equations (1). In this case, the equations of the relative motion of satellites in a formation, taking into account the figure of the Earth, will take the form:

$$\begin{aligned}
&\ddot{x} - 2n\dot{y} - 3n^2 x = -\frac{3}{2}\mu J_2 \frac{r_E^2}{R^4}\left(1 - 3\sin^2 I \sin^2 u\right), \\
&\ddot{y} + 2n\dot{x} = -\frac{3}{2}\mu J_2 \frac{r_E^2}{R^4}\sin^2 I \sin 2u, \\
&\ddot{z} + n^2 z = -\frac{3}{2}\mu J_2 \frac{r_E^2}{R^4}\sin u \sin 2I.
\end{aligned} \tag{3}$$

Since the obtained components of the force vector and the potential, taking into account J2, are functions of time, it is proposed to use the averaged value [4,5] for ∇U_{J_2}:

$$\overline{\nabla U_{J_2}} = \frac{1}{2\pi}\int_0^{2\pi} \nabla U_{J_2}(u)du = n^2 \begin{bmatrix} -3s & 0 & 0 \\ 0 & -s & 0 \\ 0 & 0 & 4s \end{bmatrix}, \tag{4}$$

where $s = \frac{3J_2 r_E^2}{8R^3}(1 + 3\cos 2I)$.

To reduce this influence disturbances from the J_2, the OCS circulation frequency must be changed. Sedwick and Schweigart [6,7] proposed the following value for the frequency:

$$n_J = nc, \quad c = \sqrt{1+s} \tag{5}$$

And the equations of motion take the form:

$$\begin{aligned}
&\ddot{x} - 2n_J\dot{y} - (5n_J^2 - 2n^2)x = \\
&= -3n^2 J_2 \frac{r_E^2}{R}\left(\frac{1}{2} - \frac{3\sin^2 I \sin^2(kt)}{2}\right), \\
&\ddot{y} + n_J\dot{x} = -3n^2 J_2 \frac{r_E^2}{R}\sin^2 I \sin 2(kt)\cos(kt), \\
&\ddot{z} + q^2 z = 2l\cos(qt + \varphi)
\end{aligned} \tag{6}$$

where $q = n_J + \frac{3nJ_2 r_E^2}{2R^2}\cos I_2, \qquad k = n_J + \frac{3\sqrt{\mu} J_2 r_E^2}{2R^{7/2}}\cos^2 I,$

I_1, I_2 is the inclination of orbits satellites in formation (it can be deputy and reference satellite). Since the orbits of satellites are close, the difference $I_2 - I_1$ is small.

2. Formation configuration control methods

Let us write the equations for the controlled motion of a satellite in a formation in the form (6):

$$\ddot{x} - 2nc\dot{y} - (5c^2 - 2)n^2 x = F_x + u_x,$$
$$\ddot{y} + 2nc\dot{x} = F_y + u_y, \qquad (7)$$
$$\ddot{z} + q^2 z = F_z + u_z + 2lq\cos(qt + \varphi),$$

where F_x, F_y, F_z are the forces caused by the J2; u_x, u_y, u_z are control forces produced by actuators (thrusters).

In this work we assumed control force as linear function:

$$u_x = -k_x(x - x_T) - k_{Vx}v_x,$$
$$u_y = -k_y(y - y_T) - k_{Vy}v_y, \qquad (8)$$
$$u_z = -k_z(z - z_T) - k_{Vz}v_z,$$

where x_T, y_T, z_T are required position of satellite in formation relative the reference coordinate system, v_x, v_y, v_z are relative velocity of satellites in formation, $k_x, k_{Vx}, k_y, k_{Vy}, k_z, k_{Vz}$, are constant coefficients.

To determine the unknown feedback $k_x, k_{Vx}, k_y, k_{Vy}, k_z, k_{Vz}$, in expressions for the control acceleration (8) we apply two methods. First method is the root method (RLM) that is based on locating the roots of characteristic equation of the system in the special areas of complex plain.

The roots of the characteristic equation of the fourth order as the roots of the Butterworth polynomial in the form:

$$\lambda^4 + 2.613\Omega_r\lambda^3 + 3.4141\Omega_r{}^2\lambda^2 + 2.613\Omega_r{}^3\lambda + \Omega_r{}^4 = 0\,. \quad (9)$$

where $\Omega_r = \dfrac{t_n}{t_p}$, t_n is the normalized time of the transient process, t_p is the real time of the transient process.

Using the parameters of the reference unperturbed orbit of the main satellite in the form: the radius of the circular orbit is 42000 km, the inclination $I = 35^0$ for the RLM we obtain following control coefficients:

$$k_x = 1.440000015961524, k_{Vx} = 1.712484599078189,$$
$$k_y = 0.707540910783830, k_{Vy} = 1.420872408915288, \quad (10)$$
$$k_z = 0.796060579653223, k_{Vz} = 1.498642992006525.$$

The second method to determine the unknown feedback coefficients $k_x, k_{Vx}, k_y, k_{Vy}, k_z, k_{Vz}$ the (8), which ensure the motion of the satellite at a certain distance from the center of the formation, it was decided to use a linear quadratic regulator, which is obtained by minimizing the quality criterion of the form:

$$J = \frac{1}{2}\int(\Delta\vec{r}^T Q\Delta\vec{r} + \vec{u}^T R\vec{u})dt, \qquad (11)$$

where $\Delta\vec{r} = \left[x - x_T, y - y_T, z - z_T, v_x, v_y, v_z\right]$, Q, R are positive matrices with constant components.

The linear quadratic performance criterion can be used for linear systems of the form:

$$\dot{\vec{X}} = A\vec{X} + B\vec{u}\ , \qquad (12)$$

where $\vec{X}$ is the state vector, A is the system matrix, B is the control matrix.

We bring equations (7) to the form (12):

$$\dot{x} = v_x,\ \dot{y} = v_y,\ \dot{z} = v_z,$$
$$\dot{v}_x = \ddot{x} - 2nc\dot{y} - (5c^2 - 2)n^2 x,\ \dot{v}_y = \ddot{y} + 2nc\dot{x},\ \dot{v}_z = -q^2 z + u_z \qquad (13)$$

For equations of the form (13), the matrices A and B have the form:

$$A = [0\,0\,0\,1\,0\,0 \quad 0\,0\,0\,0\,1\,0 \quad 0\,0\,0\,0\,0\,1$$
$$(5c^2 - 2)n^2\ 0\,0\,0\,2nc\,0 \quad 0\,0\,0 - 2nc\,0\,0 \quad 0\,0 - q^2\,0\,0\,0] \qquad (14)$$

$$B = [0\,0\,0\,0\,0\,0 \quad 0\,0\,0\,1\,0\,0 \quad 0\,1\,0\,0\,0\,1] \qquad (15)$$

The control acceleration $\vec{u} = [u_x, u_y, u_z]$], obtained as a result of minimizing the performance criterion (11) has the form:

$$\vec{u} = -R^{-1}B^T P\Delta\vec{r} = K\Delta\vec{r}\ , \qquad (16)$$

where the matrix R has the form:

$$R = 0.0001 \cdot diag[111]\,. \qquad (17)$$

And the matrix P can be determined from the equation:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0. \qquad (18)$$

$$Q = 0.0001 \cdot diag[111111]\,. \qquad (19)$$

As a result of solving equations (18), the matrix of feedback coefficients is determined using the LQR method:

$$K = [k_x\ 0\ 0\ k_{Vx}\ 0\ 0 \quad 0\ k_y\ 0\ 0\ k_{Vy}\ 0\ 0 \quad 0\ 0\ k_z\ 0\ 0\ k_{Vz}]\,, \qquad (20)$$

where

$$k_x = 1.000000012414612, k_{Vx} = 1.732050814736457,$$
$$k_y = 0.999999996453073, k_{Vy} = 1.732050805521059, \quad (21)$$
$$k_z = 0.999999994679245, k_{Vz} = 1.732050804496939.$$

3. Simulation results

To check the obtained coefficients, we will carry out a numerical solution of equations (7) under the initial conditions that determine the location of four satellites relative to their geometric center.

As the required position of the satellites in a formation x_T, y_T, z_T , we set the initial conditions given above:

$$x_{0T} = 0.54433105m, y_{0T} = 0m, z_{0T} = 0m,$$
$$x_{1T} = 0.27216552m, y_{1T} = 0m, z_{1T} = 0.54433105m,$$
$$x_{2T} = 0.27216552m, y_{2T} = -0.40824829m, \quad (22)$$
$$z_{2T} = -0.27216552m, x_{3T} = 0.54433105m,$$
$$y_{3T} = 0.40824829m, z_{3T} = -0.27216552m,$$

In this research, the volume of the formation [8] was obtained as a main quality factor of configuration stability (only for short time period).

The results of numerical studies are shown in the figures below.

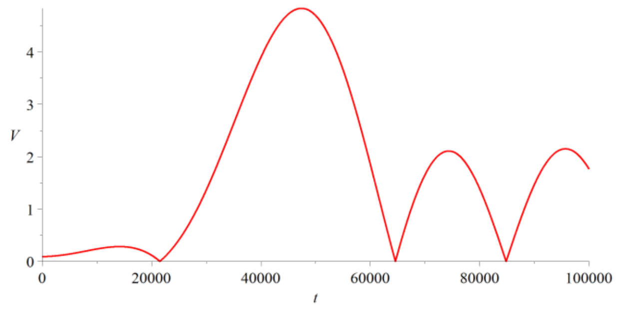

Fig. 3. Change in the formation volume in 100000 sec without control.

The results in Figure 3 show that the configuration of the satellites changes under the influence of external forces which takes into account the shape of the Earth, and approaches the initial one with some periodicity. Keeping the configuration over a long period also requires the introduction of a control action. For example, the maximum increase of the formation volume is observed at a half-cycle, i.e. at 45000-50000 sec. In this case, the volume of formation is about 5 cubic meters, which makes it impossible to complete the mission.

Figure 4 illustrates the change in the volume of the formation configuration, which already takes on sufficiently large values already at 1000 sec. Over time, the change becomes more pronounced.

As mentioned above, two control algorithms LQR and RLM were considered to save the formation configuration.

Figure 5 shows the result of changing the volume of the configuration with two control methods RLM and LQR at one time interval. It is shows that the effect of external forces caused by the shape of the Earth is leveled at the initial stage of motion and the entire subsequent period of time the formation volume remains within the required values. At the same time, it is observed that the control action based on LQR makes it possible to achieve the required position of the satellites in a configuration with a smaller overshoot compared to RLM (0.002%).

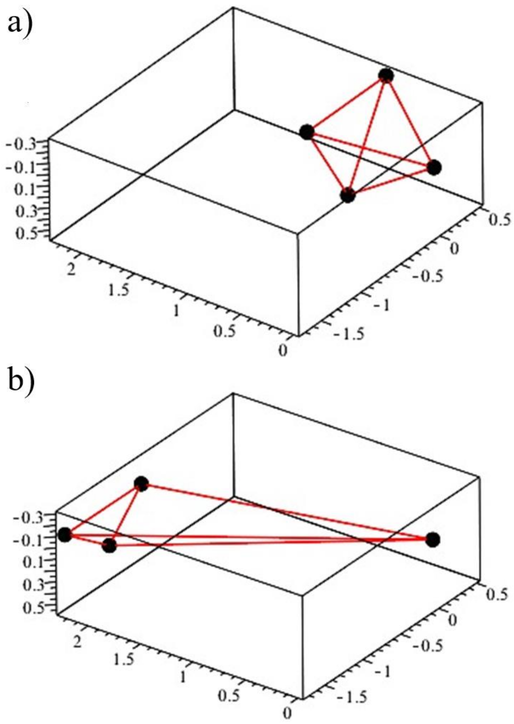

Fig. 4. Change in the formation volume without control: a) at the t=0 sec, b) at the t=1000 sec

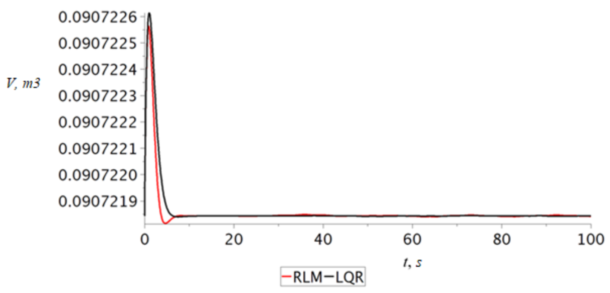

Fig. 5. Change in the volume of formation in 100 seconds during control

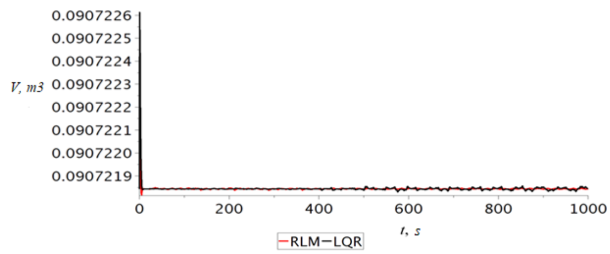

Fig. 6. Change in the volume of formation in 1000 seconds during control

Figure 6, characterizing the change in the volume of the formation over a long period of time up to 3 days, allow us to conclude that the developed algorithms for controlling the motion of satellites in a formation ensure that the configuration of a formation is maintained over a long period of time.

4. Conclusion

Thus, in this research the dynamics of changes of a tetrahedral formation volume and control algorithms for keeping follower/deputy satellites near their original positions relative to the leader/main satellite were studied. It is shown that during passive motion, a rapid increase in the volume of configuration is observed due to the uneven displacement of the satellite position relative to the reference orbit, which is associated with the inhomogeneity of the Earth's gravitational field. As an example, two modern control algorithms have been proposed for keeping satellites, which provide control over short and long periods of time. A comparative analysis of the advantages of the proposed control systems was also carried out.

Acknowledgments

Work has been supported financially by the research project № AP09260469 "Development of a control system for configuration keeping of the spacecraft formation with taking into account the uncertainties", for 2021-2023 of the al-Farabi Kazakh National University, which is gratefully acknowledged by the authors.

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