Design of Low-Thrust Control in Station Keeping Maneuver for Small Geostationary Satellites

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Abstract

The purpose of this study is simulating the station keeping maneuvers of a geostationary (GEO) satellite and applying a suitable control scheme to be able to keep the satellite in desired orbit against the external disturbance forces using an electric propulsion system. In general, the Keplerian elements are used to show the variation of the orbit over time and calculations are done with eccentricity, inclination and longitude vectors. Different from classical station keeping maneuver, relative motion is processed in this study. Through using relative motion, all required adjustments are interpreted with x, y and z with respect to the target point. To create the required control scheme, it is necessary to model the satellite's equation of motion and the motion is described with Clohessy-Hill equations as relative to virtual reference. The next step is calculation of the thrust amount to overcome the perturbation and to decrease the distance between two points.

Key words: Station-keeping, small satellites, LQR controller, relative manoeuvre, electric propulsion

1 Introduction

In order to keep the satellite rotating simultaneously with the earth, it must have an ideal circular orbit and there should not be any disturbance effects, however in reality it is not possible. During its lifetime, the satellite is exposed to the irregular gravitational force caused by the nonuniform shape of the earth and particles from the sun in the orbit. As a result of these effects, the satellite's circular orbit will be corrupted. To compensate for disturbances in the orbit when a satellite moves far from its nominal position, satellites must activate their orbit control system and perform proper maneuver in these time intervals[1].

While satellites in geostationary orbit are broadcasting to a fixed point on the Earth, they can be allowed to deviate $\pm 0.1°$ from their correct subsatellite point and ±50 km in the radial direction [2]. This limitation is defined by the International Telecommunication Union (ITU) to decrease the risk of collision and avoid the frequency interference between satellites. That specified region is called the communication window in the orbital frame. With necessary operations which are called as station keeping maneuvers, the satellite is kept in predefined borders of this window throughout its service life.

Studies related with station keeping in the literature, have been calculated over orbital parameters, and this has led to complex equations, because the orbital parameters values depend on each other and it is not easy to find out the direction of optimal maneuver to rearrange orbital parameters as desired. As a result, complex optimization calculations are required to complete station keeping maneuvers. By contrast, formation flight equations can manipulate position and velocity vectors without considering the orbital parameters. Clohessy-Wiltshire, shortly Hill's equations of motion is a method which is used to describe the relative motion between two satellites. In this study, moving the satellite from its initial point to desired point is modeled as a fictitious formation flight problem and Hill's equations used in relative flight dynamics are consulted. The real satellite is modeled as a chaser vehicle and it is re-positioned with calculated firings towards the target point as if following a virtual leader satellite. There should be a direct control over the relative position and orientation between satellites within the formation flight. Therefore, formation flight models require active, real-time and closed-loop control.

The aim of this work is to derive a time varying linearised model of nonlinear model for the dynamics of a geostationary satellite and to solve station keeping problems including all space environment disturbances. Within the

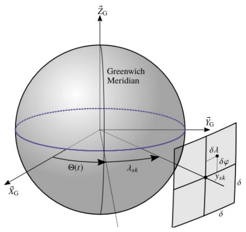

Figure 1: Communication window demonstration

scope of this study, the state of the system is established considering the earth zonal harmonic gravity model and the control algorithm is formulated as a linear quadratic regulator problem.

2 Systems Design

To eliminate effects of disturbance forces on the orbit and to keep the satellite in the defined window, thrusters can be periodically operated. Several operations and varying studies showed that an accurate station keeping maneuver of GEO satellites can be performed by electric and/or chemical systems [3].

The objective of station keeping maneuvers are maintaining the location of the satellite and keeping the vehicle inside the predefined region. The station keeping window can be defined as a box which has dimensions $2\delta X 2\delta$ in the latitude and longitude plane. In this paper, the satellite is supposed to have a constant attitude in the local orbital frame which means estimations made without considering the attitude deterioration. It is reckoned as during maneuver periods the satellite attitude is to be three-axis-stabilized using actuators and sensors.

In general, a station keeping plan is computed with an open loop control system for a predefined time period by operators on the ground and then it is uploaded onboard to execute it. Open loop control systems have some issues during the operation because they are not considering the model uncertainties [3]. Besides, ground-in-the-loop control scheme allows to compensate for perturbations and small inaccuracies of actuators in a short period of time and is controlled from ground manually so that control type is very common.

Regarding the placement of thrusters and the direction of nozzles, designers should take into account plume effect and other thermal constraints of whole systems on the satellite. To diminish plume contagion and thermal challenges, there can be used deployed pointing mechanisms such as robotic arms or gimbals to orient thrusters' nozzles and move away the thermal effects. As mentioned in [4], ESA also used articulated arms to orient thrusters during orbit raising maneuvers and now are using them to hold position. Even though electric propulsion systems have low impulse levels, moving articulated systems also help the increment of thrust efficiency in desired directions and enhance the overall system performance and satellite lifetime.

As mentioned in Introduction section within the scope of the study, station keeping maneuver is set up as a formation flight problem. In the following sections, background information related relative motion dynamics, space environment effects and used equations are basically explained.

2.1 Orbital Frames

In general for space application, satellites' motion and state vectors are described in an Earth-centered coordinate system. The measured position and velocity vectors are usually in Earth Centered Inertial (ECI) frame. The origin of the ECI frame is located at the center of the Earth. The X-axis of the ECI frame is pointing at the vernal equinox; the Z-axis is aligned with the polar axis of the Earth. The Y-axis completes the right handed orthogonal coordinate system. After all relative estimations, external forces should be calculated and added to the system dynamics with respect to the inertial frame.

Another reference frame used in the study is the Clohessy-Wiltshire (CW) to describe relative motion between virtual target point and real orbital position. In proposed work, there is one real satellite which is affected by all external forces and named as chaser satellite in terms of formation flight. The origin of the CW frame is located at the center of gravity of the virtual satellite which is always moving on a predefined circular trajectory and the satellite is named leader satellite. This leader satellite is totally fictitious and its duty is providing a reference path to transmit the real satellite to ideal in orbit position. After general navigation vectors are determined according to the inertial frame, CW reference frame is used to describe relative motion between two satellites.

Axes of the CW frame can be defined briefly as x is pointing to the radial direction of the chief satellite, z is perpendicular to the orbital plane of the chief satellite, and y completes the right handed orthogonal coordinate system.

2.2 Equation of Relative Motion

Regarding to Newton's 2nd Law, the satellite's motion can be described with two nonlinear equations,

$$\ddot{\vec{r}} = \vec{r}\dot{\theta}^2 - \frac{\mu}{r^2} + v_r \tag{1}$$

$$\ddot{\theta} = \frac{-2\dot{\vec{r}}\dot{\theta}}{r} + \frac{1}{r}v_\theta \tag{2}$$

- $\vec{r}$ is vector radius of the orbit and r is the magnitude of it,
- θ is the angle of rotation,
- μ is the Earth's gravitational constant coefficient,
- v_r and v_θ are perturbations

For an ideal case, it can be assumed that no perturbation force affects the orbit and for an ideal geosynchronous orbit, it will be circular. $\dot{\omega} = \ddot{\theta} = 0$ Regarding to specified characteristics, relative motion in a circular orbit can be easily defined with the following equations:

$$\ddot{\vec{r}} = \vec{r}\dot{\theta}^2 - \frac{\mu}{r^2} = 0 \tag{3}$$

$\vec{\rho}$ is the relative distance between two satellites and $\vec{r}_1 \approx \vec{r}_2$

$$\vec{\rho} = \vec{r}_1 - \vec{r}_2 \tag{4}$$

$$\ddot{\vec{\rho}} = \ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 = \mu\left(\frac{\vec{r}_1}{r_1^3} - \frac{\vec{r}_2}{r_2^3}\right) + \vec{f} \tag{5}$$

$\vec{r}_1$ is describing the first satellite which is not distracted by perturbations and does not have the thrust force so it is a representation of an ideal orbit. The $\vec{f}$ means all external forces which are applied on the second satellite, the actual satellite in the orbit.

If the steps are followed according to Vallado[5], last equations for near-circular orbits become

$$\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = 0 \tag{6}$$

$$\ddot{y} + 2\omega\dot{x} = 0 \tag{7}$$

$$\ddot{z} + \omega^2 z = 0 \tag{8}$$

The equations from 6 to 8 represent relative motion of the satellite and are called Clohessy-Wiltshire (CW) or shortly Hill's equations. Additionally, we can add the effects of on-board thrusters, u and disturbances, d and obtain linearized relative motion equations [6].

$$\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = u_x + d_x \tag{9}$$

$$\ddot{y} + 2\omega\dot{x} = u_y + d_y \tag{10}$$

$$\ddot{z} + \omega^2 z = u_z + d_z \tag{11}$$

ω is the constant angular velocity of the circular reference orbit. The state-space form can be formulated as below

$$\ddot{\vec{x}} = A\vec{x} + B\vec{u} + F\vec{d} \tag{12}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3\omega^2 & 0 & 0 & 0 & 2\omega & 0 \\ 0 & 0 & 0 & -2\omega & 0 & 0 \\ 0 & 0 & -\omega^2 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/m_s & 0 & 0 \\ 0 & 1/m_s & 0 \\ 0 & 0 & 1/m_s \end{bmatrix} \tag{14}$$

$\vec{x}$ is the state of the satellite, $B\vec{u}$ matrix represents the acceleration created by thrusters on the satellite so the unit of input $\vec{u}$ is in Newton [N] and $F\vec{d}$ expresses the acceleration created by disturbances.

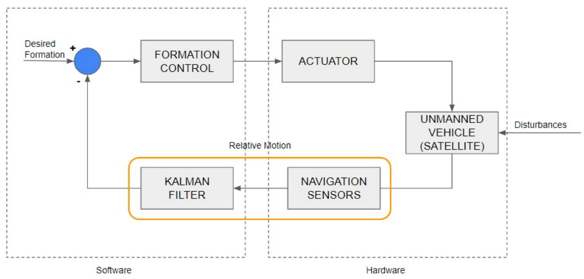

Figure 2: Architecture of formation flight control model [7]

2.3 Orbit Determination

To start orbit maintenance maneuvers, position of the satellite should be specified accurately hence there is a process to make the best estimation of satellites' position over time using raw measurement values and it is called as orbit determination. This estimation includes finding the correct variables to describe the trajectory processing related information. To acquire observation data, some sensors can be used such as magnetometers, Global Navigation Satellite Systems (GNSS) receivers, or tracking antennas on the Earth. Orbit determination problem is easy when the GNSS signals are available but for some missions such as high Earth orbits like geostationary orbit or interplanetary missions, GNSS signals are not feasible. In these cases, estimations are made by ground base antenna systems. In general, for navigation and maneuver implementation, meter level orbit determination accuracy is desirable. To succeed at this accuracy, there are proposed combined methods in literature for example Guo et al. issued (2010) a new strategy for GEO satellites precise orbit determination (POD). They reported the combination of satellite laser ranging (SLR) and C-band ranging data for the POD algorithm. In their work, bias coming from C-band ranging is calibrated with SLR measurement and POD accuracy meets sub-meter accuracy requirements for GEO satellites[8].

In POD algorithms, Kalman filter is usually used to process measured tracking data. The process of orbit estimation of satellites by using antenna tracking data is nonlinear so EFK (extended Kalman filter) or UFK (unscented Kalman filter) can be chosen [9].

In the proposed study, it is assumed that a combined orbit determination technique is used with EFK and POD accuracy is estimated at sub-meter level in three directions. This uncertainty which comes from estimation is modeled in the Simulink environment.

2.4 Controller Design

In the scope of this work, Linear Quadratic Regulator is developed which is basically minimizing the cost function of a time varying system. The plant of the satellite can be described as a discrete time variant system that will be controlled by the linear equations below. The relative vector in the model is taken as an input to the control part.

$$\vec{x}(t+1) = \mathbf{A}(t)\vec{x}(t) + \mathbf{B}(t)\vec{u}(t), \vec{x}(0) = \vec{x_0} \tag{15}$$

The main idea of the optimal control is to find optimal control input, $\vec{u}(t)$, which drives the system along optimal path $\vec{x}(t)$, while cost function is minimized. Let us define the time interval of interested behavior of the system as $[i,N]$ where N is specifying the final state time and $\vec{x_n}$ is for the final state of the system. L_k is a time varying function of the state.

$$J_i = \phi(N, \vec{x_N}) + \sum_{k=1}^{N-1} L_k(\vec{x_k}, \vec{u_k}) \tag{16}$$

The purpose of this study is to find optimal control input to decrease relative distance between two satellites and minimize the duration of maneuver. If the 16 is written in terms of weighting matrices such as Q, R and S

$$J_i = \frac{1}{2}\vec{x}_N^T S_N \vec{x}_N + \frac{1}{2}\sum_{k=1}^{N-1}(\vec{x}_k Q_k \vec{x}_k + \vec{u}_k^T R_k \vec{u}_k) \tag{17}$$

where the Q_k, R_k and S_k are symmetric positive semidefinite matrices. To solve the LQR problem, the well known Riccati equation involves the solution approach as given in the 18 and K_k is the gain of the Riccati equation.

$$S_k = A_k^T(S_{k+1}^{-}1 + B_k R_k^{-} 1 B_k^T)A_K + Q_k \tag{18}$$

$$\vec{u}_k = -R_k^{-} 1 B_k^T S_{k+1} \vec{x}_{k+1} \tag{19}$$

$$\vec{u}_k = -K_k \vec{x}_k \tag{20}$$

2.5 External Forces

During nominal operation, satellites are affected by several external forces. These forces express the specific force or acceleration that deforms the orbit of a satellite. We can split external forces into two categories as controllable and uncontrollable. Controllable external forces are applied intentionally to the system such as thrust force. Uncontrollable forces are mainly environmental effects which totally depend on the external objects and are commonly called disturbance forces. Disturbance forces are occurred by such as solar pressure, atmospheric drag or gravitational perturbations and affected by physical characteristics of satellites. Intensity of environmental effects vary according to the operation environment of the satellite. Therefore, the type and magnitude of disturbances change with the operation. For instance, while atmospheric drag force is the strongest effect in low earth orbits, it loses its importance in geostationary operations because of the insignificant atmospheric density. Following the study, external forces that influence the geostationary satellites are given with their mathematical models.

2.5.1 Gravitational Effects

Due to the Earth's imperfect spherical shape and non-homogeneous mass distribution, the gravitational force acting on the satellite varies throughout the rotation. Within the variation of the gravitational force, a satellite cannot follow the exact path that is represented in two-body propagation equations.

To model this varying mass distribution of the Earth, it is split in zonal harmonics and corresponds to division zonal coefficients are changing. J_2 has the most significant effect and it is 1000 times larger than the closest zonal coefficient, J_3 [6] so in general, higher degrees of zonal coefficients are not taken into account. In the ordinary way, the Hill's equations mentioned in Equation of Relative Motion section does not include any zonal harmonics coefficient. As specified by [10], typical Hill's linearized equations are not enough to cover effects of J_2 disturbance force on the satellite. Therefore Sedwick and Schweighart [10] defined a new set of equations which are similar to original form of the Hill's equations but they have a constant coefficient to produce well-suited model for satellite relative motion which includes the J_2 effects [11].

$$\ddot{x} - 2c\omega\dot{y} - (5c^2 - 2)\omega^2 x = f_x \tag{21}$$

$$\ddot{y} + 2c\omega\dot{x} = f_y \tag{22}$$

$$\ddot{z} + (3c^2 - 2)\omega^2 z = f_z \tag{23}$$

$$c = \sqrt{1 + \frac{3J_2\mu R_E^2}{2r_{ref}^2}(1 + 3\cos 2i_{ref})} \tag{24}$$

2.5.2 Solar Radiation Pressure

Solar radiation pressure is another disturbance source for geostationary satellites. To model solar radiation precisely, solar cycles and variations should be considered and modeled accurately. Additionally, the cross-sectional area of the satellite which will be exposed to solar radiation pressure should be estimated in compliance with the orientation of the space vehicle [5]. As mentioned in [5], solar radiation pressure is formulated using reflectivity coefficient c_R, solar radiation pressure p_{srp} and the exposed area A.

$$\vec{F_{srp}} = -p_{srp}c_R A \frac{\vec{r}_{Sun_{ECI}}}{|\vec{r}_{Sun_{ECI}}|} \tag{25}$$

A solar radiation constant often called the intensity, irradiance, or solar flux is $SF = 1367W/m^2$ and the Solar pressure p_{srp} is calculated by dividing the solar flux by the speed of light, c

$$p_{spr} = \frac{1367}{3x10^8}\frac{N}{m^2} \tag{26}$$

2.5.3 Third-body Perturbations

Third-body is the general expression of large celestial bodies such as the Sun and Moon. Especially, the third body's effect becomes strong when the drag force begins to diminish. These bodies create a greater effect on geostationary satellites [5]. Basically, the acceleration created by third-bodies can be formulated as below

$$\ddot{\vec{r}}_{E/sat} = -\frac{\mu_E \vec{r}_{E/s}}{r_{E/s}^3} + \mu_3\left(\frac{\vec{r}_{sat/3}}{r_{sat/3}^3} - \frac{\vec{r}_{E/3}}{r_{E/3}^3}\right) \tag{27}$$

E is used to symbolize the Earth. $\vec{r}_{E/sat}$ is the vector between the center of Earth and satellite's center of gravity. In the same manner as previous, the third-body is symbolized by 3.

2.5.4 Effects of Thrusters

After formation is modeled, orbit slowly but continuously changes by means of thrust force. However, before starting maneuver, precise orbit determination and active attitude control is required to achieve maneuver as planned. Orbit determination method is described in Orbit Determination section. For attitude control, it is assumed that the deterioration produced by thrusters should be actively controlled. The attitude of the satellite is measured by sensors and the deterioration is suppressed by reaction wheels and formation maneuvers are accomplished by activating the thrusters. The value of the thrust range is related to the mission requirements and the designed propulsion system of the satellite. Typically, low range thrusts are selected in formation flights. In this study, it is assumed that the modeled satellite is equipped with the six xenon ion thrusters in the propulsion system and each thruster will be 0.165N [12].

While thrusters are firing, they apply a certain acceleration to the satellite which increases the velocity (Δv) in that time interval(Δt) and some time later, it causes a position change (Δr) and this period can be called as maneuver time.

$$\Delta v = \frac{-F\Delta t}{m_0} \tag{28}$$

m_0 is the satellite mass and $F\Delta t$ is the thrust transferred by the actuator.

3 Simulation and Results

It is assumed that in real life the satellite's position is precisely determined with the mentioned method which is reported by Guo et al. [8] and the distance between the ideal point which is targeted as the virtual leader satellite's location estimated with the orbit propagator is calculated. In the simulation, the initial distance is 0.4 km. Using estimated position data, linearized equations of motion for the satellite are computed and state-space matrices mentioned in section are obtained. State-space matrices are inserted in the estimation of LQR gain. In the designed relative navigation architecture, the LQR gain is obtained by the manual tuning method and estimated gain is entered into the LQR Controller block of the Simulink model, in Figure3.

In the Simulink model, X represents the state of the satellite. The Eutelsat 115 West B satellite is taken as a model to define parameters of the satellite and the thruster, and its features are presented in the table below [12].

The following differential equation 29 governs the motion of a satellite around the Earth and shows the external forces acting on the vehicle. It is a basic expression of the

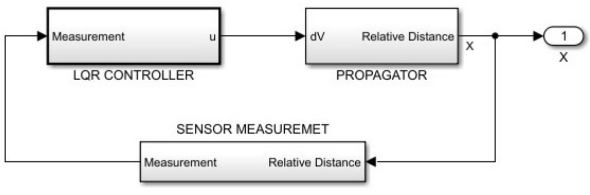

Figure 3: Simulink diagram

Satellite mass	2205.0 kg
Propulsion System	6 x Xenon Ion Thruster (XIPS)
Thrust	0.165 N
Propellant	Xenon
Specific Impulse	3500 s

Table 1: Parameters of designed satellite [12]

system of equations in a Cartesian coordinate system that originates at the center of the Earth and is solved by means of the fourth order Runge Kutta method. It is an iterative method to numerically approximate solutions of ODE's [12].

$$\frac{d^2\vec{r}}{dt^2} = -\frac{\mu}{r^3}\vec{r} + \vec{a}_T + \vec{a}_{Dist} \tag{29}$$

The acceleration provided by the thrusters of the spacecraft is defined as follows.

$$\vec{a}_T = \frac{d\vec{v}}{dt} = \frac{\vec{T}}{m} \tag{30}$$

$\vec{r}$ is the position vector, the term $\vec{a}_T$ corresponds to acceleration provided by thruster.

In the designed relative flight architecture, the value obtained from the satellite dynamic model is taken as the reference state for follower satellite X_t. The output of the measurement block which includes the Kalman filter solution is considered as the feedback value H_t. During the simulation, the controller aims to diminish the difference between dynamic model outputs and measurement values.

Relative distance which changes over time due to applied thrust force is given in the following figure.

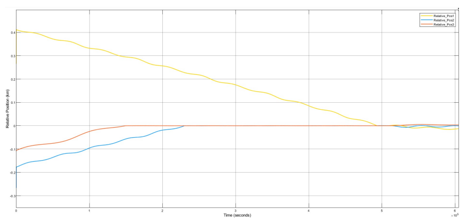

Figure 4: Variation of relative position of the chaser satellite

Within the control architecture created for the fictitious formation problem, required Δv to follow leader satellite as desired is shown in below figure.

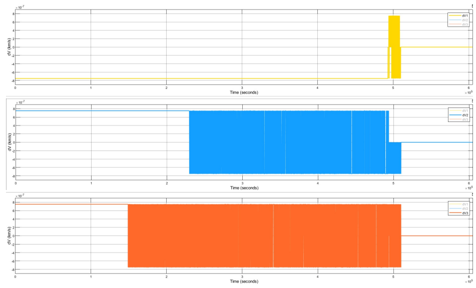

Figure 5: Estimated Δv outputs of the simulation

To reach the desired location within the defined margin, overall burning duration is calculated as 141.6 hours. The distance starts with 0.4 km and at the end of the burning section, the distance is obtained as 4.7 meters. While performing station keeping maneuver, the propulsion system actively overcomes the instantaneous disturbances and measurement uncertainties. The total mass used in the maneuver is calculated as 7.5e-04 kg.

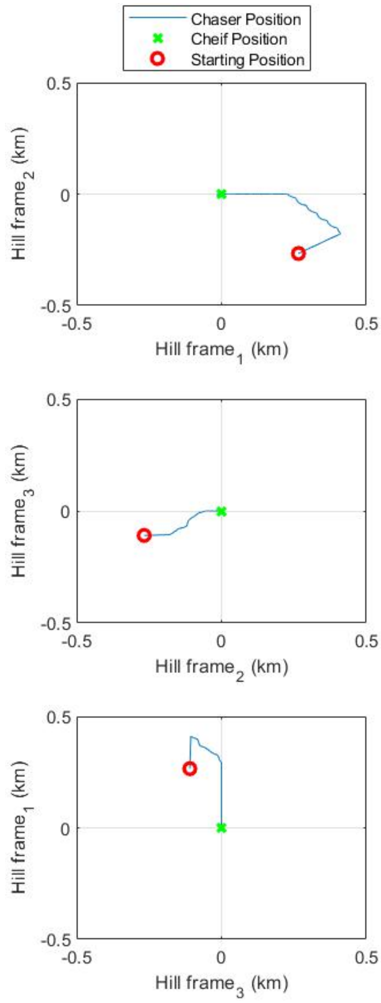

Figure 6: Variation of chaser satellite's position in communication window

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